Random Variables, Distributions, Indicators

1 Intro

1. What is the definition of a random variable?

2. If X is a random variable, what does $f(X)$ mean?

3. What is the definition of expectation and variance? Can you interpret these in terms of the distributions of random variables?

4. Give an example of something (in real life) that has a:

   (a) bernoulli distribution

   (b) binomial distribution

   (c) geometric distribution

   (d) poisson distribution

5. What distribution does an indicator random variable have?

6. Show that for any indicator variable X, $E[X] = E[X^2]$.

2 Problems

1. You roll a six-sided balanced die until you get the first 6. How many dots do you accumulate, on average, including the 6 on the last roll?

2. (a) You want to collect 20 Marvel superheroes, which come randomly in cereal boxes. How many cereal boxes do you expect to buy before you collect them all?

   (b) Now let's say you can only buy 20 cereal boxes. What's the expected number of superheroes you'll get?

3. Let's look at completely random n-length binary sequences where $n \geq 2$. We want to count how many 'double-ones' we get in the sequence, i.e. the sequence 011101101 has 3 'double-ones'.

   (a) What's the expected value of number of double-ones we get for a sequence of length n?

   (b) What's the variance?