Continuous Probability, Joint Distributions

1 Intro

1. How do continuous variables differ from discrete variables?

2. How do we take expectation and variance for continuous variables?

3. What are the analogs of the following distributions for continuous random variables?

   (a) Uniform distribution

   (b) Geometric distribution

   (c) Binomial distribution

4. What are the properties of the CDF? Of the PDF? How do we get one from the other?

5. Show how we can write every normal distribution in terms of the standard normal $N(\mu = 0, \sigma^2 = 1)$.

2 Problems

1. Let X be a random variable with pdf given by

$$f_X(x) = \begin{cases} cx^2 & \text{if } |x| < 1 \\ 0 & \text{otherwise} \end{cases}$$

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   (a) Find c that makes this a valid random variable.

(b) Find $E[X]$ and $var[X]$.

(c) Find $P(X \leq \frac{1}{2})$.

2. Let X be a random variable with pdf

$$f_X(x) = \begin{cases} 4x^3 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X \leq \frac{2}{3} \mid X > \frac{1}{3})$.

3. Two real numbers are chosen uniformly from $[0, 1]$. What is the probability that their sum is less than or equal to 1 given that one of them is less than or equal to 1/2?

4. Let X and Y be jointly continuous r.vs. with joint PDF

$$f_{X,Y}(x, y) = \begin{cases} 6e^{-(2x+3y)} & \text{if } x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

. Are X and Y independent? Find $P(X > Y)$.

5. Let X be a positive continuous r.v. Show that $E[X] = \int_0^\infty P(X \geq x)dx$.