Continuous and Joint Distributions Practice

I chose to pick a lot of midterm and final problems for practice this week. Some of these are quite challenging, but interesting!

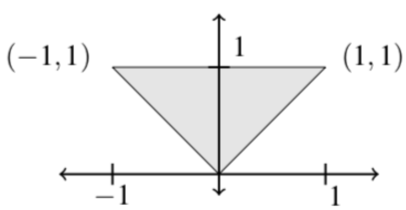
1. (Spring 2017, 5.9) What is the probability density function for a continuous random variable with $Pr(X \leq x) = 1 - \frac{1}{x}$ for $x \geq 1$ and $Pr(X \leq x) = 0$ for $x < 1$?

2. (Fall 2017, 6.10) X and Y are continuous random variables and are uniformly distributed with pdf $f(x, y) = c$ over their region of support (the area where they have even a chance of taking on values). Their region of support is the following: $\{1 < X < 2, 1 < Y < 4\} \cup \{2 < X < 3, 2 < Y < 3\}$.

   (a) Find c. Are X and Y independent?

   (b) Find the marginal distributions of X and Y.

3. (Spring 2017, 7.1) Consider a point (x, y) is chosen uniformly from the area below:



   Are X and Y independent? What is $Pr(Y > x)$? $E[X]$? $E[Y]$?

4. (Fall 2017, 6.4) Let $X_1, \ldots X_n$ be i.i.d $U[0, 1]$ random variables.

   (a) Find the PDF of $Y = \min(X_1, \ldots, X_n)$.

(b) Let $Z = \max(X_1, \ldots, X_{100})$. What is $E[Z]$?

5. (Fall 2017, 6.13) Let $X \sim \text{Exp}(\lambda)$, and let $\lceil X \rceil$ denote the ceiling of X (the smallest integer greater than or equal to X). Find the distribution of $\lceil X \rceil$. Does it look like a distribution we've seen before? What parameters?

6. (Fall 2017, 6.14) Let $X \sim N(0, 1)$ and $Y \sim N(1, 1)$ be independent Gaussian random variables. You get an observation $z = 0.6$ that is equally likely to come from X and Y. You want to decide whether z came from X or Y by evaluating which decision leads to a larger probability of being right.

   (a) If you decide z came from X, what is the probability you are right?

   (b) Should you decide z is from X or from Y to get a larger probability of being right?

7. (Spring 2017, 7.2) You pick a real number from the range [0, 1] using the uniform distribution. Then your friend independently picks a real number uniformly at random from the range [0, 2].

   (a) What is the probability that your two numbers differ by no more than one?

   (b) Now you pick a variable in the range [0, 1] with pdf $f(x) = 2x$. Then your friend still picks a real number uniformly at random from [0, 2]. Now what is the probability that your two numbers differ by no more than one?