Inequalities, Estimation, Markov Chains (?)

1 Inequalities

1. What is Markov's inequality? How do you prove it?

2. What is Chebyshev's inequality? How do you prove it?

3. In what situations can we use the Central Limit Theorem? What does it tell us? Does it give us a definitely true bound?

4. Let X be the sum of 20 i.i.d geometric random variables $X_1, \ldots X_{20}$ with parameter $p = \frac{1}{2}$. Find an upper bound of $P(X \geq 26)$ using:

   (a) Markov's inequality:

   (b) Chebyshev's inequality:

5. I want to take a student pool to find the popularity of CS 70 (assume each student independently likes it with probability p), and I need to pay each student \$1 to get their opinion. Suppose I want to estimate p within 1 percent accuracy with a 95% confidence level, I want to find how much money I need to find my estimate.

   (a) What estimator could you use for p from a set of samples, $X_1, X_2, \ldots, X_n$? (it should have expectation p).

   (b) What is an upper bound on the variance of your estimator that does not depend on p?

   (c) How much money would I need to spend if I use the CLT?

   (d) How about if I use the Chebyshev bound?

2 Estimation

1. How do we define covariance of random variables? How is this related to independence?

2. Let $X \sim \text{Expo}(3)$, then define $Y \sim \text{Poiss}(X)$.
   (a) What is $E[Y|X]$? How can we find $E[Y]$?

   (b) What is $E[X|Y]$?

   (c) How do we find $cov(X, Y)$? Do we expect it to be positive or negative?

   (d) How do we find $L[Y|X]$? Which is smaller, $E[(Y - E[Y|X])^2]$, or $E[(Y - L[Y|X])^2]$?

3. Some T/F:
   (a) If X_1 and X_2 are i.i.d Expo(1) random variables, $\text{cov}(\min(X_1, X_2), \max(X_2, X_2)) = 0$.

   (b) If $X \sim \text{Geom}(p)$, then $E[X + m|X > n] = m + n + E[X]$.

3 Markov Chains???

Yes, we haven't talked about Markov chains in 70 yet, so I don't expect you to know any of this. We will talk about this just to introduce some concepts.

1. You flip a fair coin repeatedly until you get 2 heads in a row or 2 heads in 3 tosses. We wish to find the expected number of tosses you need before you stop.
   (a) Draw a Markov chain corresponding to this process, where the goal state is terminal (we cannot leave it).

   (b) What is the expected number of tosses you need to stop? Just set up the equations.